Analytic Approach of Internet Pricing Scheme Model Based on function of Bandwidth Diminished with Increasing Bandwidth

Hartati Hartati1,*, Fitri Maya Puspita2 and Indrawati Indrawati2

1 Open University, Palembang South Sumatera Indonesia
2 Sriwijaya University, Inderalaya Ogan Ilir South Sumatera Indonesia

Abstract—Along with the changing times, the internet is becoming essential for everyday life. As the internet service providers, ISPs are required to provide a good service so as to give satisfaction to consumers at a low price. However, ISPs are expected to not only take into account customer satisfaction, but also take into account the advantages gained by considering all the factors that exist. Therefore, ISPs are given the option of pricing schemes, namely flat fee, usage-based, and two part tariff pricing schemes to be applied to the utility function as a function of bandwidth decreases with increasing bandwidth to maximize the benefits ISPs with regard to the level of user satisfaction. This study analyzed the two types of customers, namely homogeneous and heterogeneous consumers. Consumers are divided into heterogeneous consumer of willingness to pay (high end and low end) and with different consumption levels (high demand and low demand). In the case of consumers with homogeneous and heterogeneous consumer of willingness to pay (high end and low end), optimal pricing scheme is obtained if the ISP uses a flat fee and a two-part tariff schemes. As for heterogeneous consumers with different consumption levels (high demand and low demand), the scheme of two part tariff is the optimal scheme to generate maximum profits.

Keywords—Pricing scheme; Flat fee; Usage based; Two-part tariff; Optimal solution.

I. INTRODUCTION

Over time, the development of technology with the development of internet services is in balance. Internet Service Provider (ISP) is required to compete to provide maximum service but at minimum cost, because at minimum cost will increase consumer interest to use the service offered. However, as the company's internet service provider, ISPs also have to consider the profits and maintaining the quality of the service with minimum prices for consumers.

ISPs are trying to provide the best service with the best pricing scheme anyway. However, there is now a wide variety of user demand internet and various applications that make the Internet providers must take into account the quality of service (Quality of Service, QoS). Basically, QoS makes it possible to provide better service to a particular request. In demonstrating efficiency in service, it is a must for ISP to show interaction between price and QoS [1].

Not only ISP, there should be a separate consideration for consumers who use the internet service there must be compatibility between the prices given to customer satisfaction. As customer satisfaction can be viewed by using the utility function. Utility function describes the level of customer satisfaction in consuming a product [2]. Internet Service Provider (ISP) is currently handling a high demand to promote good quality information. But a new initiative to develop a new price that involves QoS is only a few [3–5]. Therefore, it is important to consider the ISP customer satisfaction in creating a scheme that will make the relationship between user satisfactions with the QoS provided by ISPs.

By taking into account all the factors of the providers and consumers then, three types of pricing schemes are most often used, namely a flat fee, usage-based, and the two-part tariff [6, 7] will be further analyzed. Analysis of these strategies is divided into two parts that considers homogeneous and
heterogeneous customers. In the case of a homogeneous, all customers have the same utility on the level of consumption per day whereas in the case of heterogeneous, customers have two segments according to willingness to pay (willingness to pay) and the level of consumption (consumption level). Different analysis using different utility function has been introduced to show the variety of utility function applied to obtain new sight on choosing the satisfaction of the user both analytically or iteratively [8, 9]. Furthermore, in the article, a modification of the other utility function except Cobb-Douglas utility function will be discussed. This research will seek better schemes by using a utility function as a function of bandwidth which decreases with increasing bandwidth since this utility function is the best utility function which deals with the various level of QoS [10-12].

II. RESEARCH METHODOLOGY
The steps undertaken in this study are as follows:
1. Formulate a utility function as a function of bandwidth which decreases with increasing bandwidth (functions of bandwidth diminished with increasing bandwidth) based on three types of pricing schemes which are flat fee, usage-based, and the two-part tariff for two types of customers, namely homogeneous and heterogeneous consumers.
2. Conduct an analysis of the utility function selected in order to obtain a model scheme based on the utility function.
3. Comparing the schematic model derived from the analysis.
4. Make the appropriate conclusions and comparative analysis obtained.

III. RESULT AND DISCUSSIONS
In next steps, we define the parameter and variables involved as follows.

\[ Y_i^* = \] The consumption level of customer service \( i \) at peak hours
\[ Y_i(P_x, P_y, P) \] : The variable \( i \) consumer purposes
\[ Z_i^* = \] with respect to its participation
\[ Z(P_x, P_y, P) \] : The utility function of consumer \( i \) with \( X_i \) is the level of service usage during peak hours and \( Y_i \) is the level of service usage when not busy hours

In this case utility function as a function of bandwidth which decreases with increasing bandwidth of [11] will be applied:

\[ U_{kj} = U_{0j} + W_j \ln \frac{x_{kj}}{l_{mj}} \] (1)

Class \( J \) is divided into classes at busy times (\( x \)) and a class that is not at peak hours (\( y \)) thus obtained:

\[ U_{xx} = U_{0x} + W_x \ln \frac{x_{xx}}{l_{mx}} \] (2)
\[ U_{ky} = U_{0y} + W_y \ln \frac{x_{ky}}{l_{my}} \] (3)

with

\[ U_0 = U_{0x} + U_{0y} \] (4)
\[ W_x = a \) and \( W_y = b \]
\[ X_{xx} = X \) and \( X_{ky} = Y \]
\[ L_{mx} = X_m \) and \( L_{my} = Y_m \]

So it will become as follows.

\[ U(x, y) = U_{0x} + W_x \ln \frac{x_{xx}}{l_{mx}} + U_{0y} + W_y \ln \frac{x_{ky}}{l_{my}} \] (5)

\[ U(x, y) = U_0 + a \ln \frac{x}{x_m} + b \ln \frac{y}{y_m} \] (6)

With a few changes to simplify the calculation, then Eq. (5) is changed to:

\[ U(x, y) = U_0 + a \ln \frac{x}{x_m} + b \ln \frac{y}{y_m} \] (6)

This change was made to simplify the calculation when the minimum consumption level, \( X_m \) and \( Y_m \) and the level of consumption during peak hours and off-peak hours respectively, \( X \) dan \( Y \) can produce a minimum value of 0 rather than making value becomes negative.

A. Homogeneous Consumer

In the case of homogeneous consumers, it was assumed that all consumers have the same level of satisfaction and the maximum the same level of use in peak hours and in off-peak hours, i.e. \( \bar{X} \) and \( \bar{Y} \).

The optimization of consumer problem will be as follows:

\[ X_i^* = \] The level of customer service \( i \) consumption during peak hours
\[ X_i(P_x, P_y, P) \] : The variable \( i \) consumer purposes

\[ Z_i^* = \] with respect to its participation
\[ Z(P_x, P_y, P) \] : The utility function of consumer \( i \) with \( X_i \) is the level of service usage during peak hours and \( Y_i \) is the level of service usage when not busy hours

\[ Y_i^* = \] The consumption level of customer service \( i \) at peak hours
max \( U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}} - P_X - P_Y - P_Z \) 

(7)

Subject to

\( X \leq \bar{X}Z \) 

(8)

\( Y \leq \bar{Y}Z \) 

(9)

\( U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}} - P_X - P_Y - P_Z \geq 0 \) 

(10)

\( Z = 0 \) or 1

The optimization of supplier problem will be as follows:

\[
\max_{P_X, P_Y} \sum_i (P_X X_i^* + P_Y Y_i^* + P Z_i^*)
\]

(11)

with \((X^*, Y^*, Z^*) = \text{argmax} U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}} - P_X X - P_Y Y - P Z\)

(12)

subject to

\( X \leq \bar{X}Z \) 

(13)

\( Y \leq \bar{Y}Z \) 

(14)

\( U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}} - P_X X - P_Y Y - P Z \geq 0 \) 

(15)

\( Z = 0 \) or 1

The selection of the best scheme is determined by analysis of derivatives as follows:

**Lemma 1:** If ISPs use flat fee pricing scheme, the fee paid will be \( U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}} \) and the maximum profit will be \( \sum_i (U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}}) \).

**Proof of Lemma 1**

If the ISP chooses to use a flat fee pricing scheme then \( P_X = 0, P_Y = 0 \), and \( P = 0 \). Eq. (7) will be as follows:

\[
\max_{x, y, z} U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}} - P_X X - P_Y Y - P_Z
\]

(16)

In order to keep the maximum level of customer satisfaction then the upper limit of Equation (16) is so \( P = U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}} \). Then the fees charged on a flat fee pricing scheme is set at \( U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}} \) and the maximum profit will be \( \sum_i (U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}}) \).

**Lemma 2:** If ISPs use usage-based pricing scheme, the optimal price is \( P_X = \frac{a}{x_1} \) and \( P_Y = \frac{b}{y_1} \) with the maximum profit of \( \sum_i (a \left(1 - \frac{1}{x_1}\right) + b \left(1 - \frac{1}{y_1}\right)) \).

**Proof of Lemma 2**

If ISP chooses usage based pricing scheme then \( P_X > 0, P_Y > 0 \), dan \( P = 0 \). So Eq. (7) will be

\[
\max_{x, y, z} U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}} - P_X X - P_Y Y - P_Z
\]

(17)

To maximize Eq. (17), ISP should lower the value of \( P_X \) and \( P_Y \). Firstly, to optimize the consumer problem, we derive Eq. (17) to \( x \) and \( y \).

\[
\frac{a}{x^* + 1} - P_X = 0 \iff \frac{a}{x^* + 1} = P_X \text{ then } X^* = \frac{a}{P_X} - 1
\]

(18)

\[
\frac{b}{y^* + 1} - P_Y = 0 \iff \frac{b}{y^* + 1} = P_Y \text{ then } Y^* = \frac{b}{P_Y} - 1
\]

(19)

The optimization of supplier problem will be:

\[
\max_{P_X, P_Y} \sum_i (P_X X_i^* + P_Y Y_i^*) = \max_{P_X, P_Y} \sum_i (a - \frac{a}{x^*_i} + b - \frac{b}{y^*_i})
\]

(20)

Since \( X \) and \( Y \) are restricted so \( X^* \) and \( Y^* \) will be \( \bar{X} \) and \( \bar{Y} \). In other words, \( P_X \) and \( P_Y \) will be \( P_X = \frac{a}{x_1} \) and \( P_Y = \frac{b}{y_1} \) with maximum profit of \( \sum_i (a - \frac{a}{x_1} + b - \frac{b}{y_1}) \).

**Lemma 3:** If ISP applies two part tariff pricing scheme, the optimal price will be \( P_X = \frac{a}{x_1} \) and \( P_Y = \frac{b}{y_1} \) with \( P = U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}} - a \frac{x}{x_1} - \frac{b y}{y_1} \) with the maximum profit of \( \sum_i (U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}}) \).

**Proof of Lemma 3**

If ISPs chooses two part tariff pricing scheme, then the requirements are \( P_X > 0, P_Y > 0 \), and \( P > 0 \). Eq. (7) will be

\[
\max_{x, y, z} U_0 + a \ln \frac{x_1}{x_{m+1}} + b \ln \frac{y_1}{y_{m+1}} - P_X X - P_Y Y - P_Z
\]

(21)

We derive the Eq. (21) toward \( x \) and \( y \) to maximize \( P_X \) and \( P_Y \) so it will be Eq. (18)-(Eq.19).

The optimization of supplier problem will be as follows.
\[
\max_{P_x,P_y} \sum_i (P_x X_i^* + P_y Y_i^* + P Z_i^*) = \max_{P_x,P_y} \sum_i \left( P_x \left( \frac{a}{P_x} - 1 \right) + P_y \left( \frac{b}{P_y} - 1 \right) + P \right) 
= \max_{P_x,P_y} \sum_i \left( a - P_x + b - P_y + P \right) 
\]
(22)
since \( X \) and \( Y \) are restricted then, \( X^* \) and \( Y^* \) will be \( \bar{X} \) and \( \bar{Y} \). \( P_x \) and \( P_y \) will be \( P_x = \frac{a}{\bar{X}} \) dan \( P_y = \frac{b}{\bar{Y}} \) dan \( P = U_0 + \frac{a}{\bar{X}} \frac{x+1}{\bar{X} + 1} + b \frac{1}{\bar{Y} + 1} \ln \frac{1}{\bar{Y}} - \frac{a \bar{X}}{\bar{X} + 1} - \frac{b \bar{Y}}{\bar{Y} + 1} \). So the prices charged according to usage of the consumer will be \( \left( 1 - \frac{1}{\bar{X}} \right) + b \left( 1 - \frac{1}{\bar{Y}} \right) = \frac{a \bar{X}}{\bar{X} + 1} + \frac{b \bar{Y}}{\bar{Y} + 1} \) with the maximum profit of \( \sum_i \left( U_0 + \frac{a}{\bar{X}} \frac{x+1}{\bar{X} + 1} + b \frac{1}{\bar{Y} + 1} \ln \frac{1}{\bar{Y}} \right) \).

Then the type of flat fee and a two-part tariff pricing schemes generate huge profits equally according to maximize profit-making for the ISP (supplier) while the usage-based pricing scheme is different from the two schemes.

B. Heterogeneous Consumer: High-End and Low-End

Lemma 4: If ISP applies flat fee pricing scheme then the price set up to consumers will be \( U_0 + a_2 \ln \frac{x+1}{\bar{X} + 1} + b_2 \ln \frac{1}{\bar{Y} + 1} \) with the maximum profit of \( (m + n) \left( U_0 + a_2 \ln \frac{x+1}{\bar{X} + 1} + b_2 \ln \frac{1}{\bar{Y} + 1} \right) \).

Proof of Lemma 4

Applying flat fee scheme, then we have \( P_x = 0, P_Y = 0 \), dan \( P > 0 \). If the consumers choose to join the services, then the maximum level of satisfaction can be determined by choosing the level of consumption of \( X_1 = \bar{X}, Y_1 = \bar{Y} \) or \( X_2 = \bar{X}, Y_2 = \bar{Y} \). This makes ISP can give the prices for each high-end consumer of no more than \( U_0 + a_1 \ln \frac{x+1}{\bar{X} + 1} + b_1 \ln \frac{1}{\bar{Y} + 1} \) and low end consumer of no more than \( U_0 + a_2 \ln \frac{x+1}{\bar{X} + 1} + b_2 \ln \frac{1}{\bar{Y} + 1} \). So, it is easier to assume that \( a_1 > \frac{m+n}{m} a_2 \) and \( b_1 < \frac{m+n}{m} b_2 \). Then, ISP will set the price of \( U_0 + a_2 \ln \frac{x+1}{\bar{X} + 1} + b_2 \ln \frac{1}{\bar{Y} + 1} \) to high-end consumers and low-end consumers make profit to ISP for \( (m + n) \left( U_0 + a_2 \ln \frac{x+1}{\bar{X} + 1} + b_2 \ln \frac{1}{\bar{Y} + 1} \right) \).

Lemma 5: If ISP applies the usage based pricing scheme, then the optimal price will be \( P_x = \frac{a_2}{\bar{X}} + 1 \) and \( P_y = \frac{b_2}{\bar{Y}} + 1 \), the maximum profit is \( (m + n) \left( \frac{a_2 x + 1}{\bar{X}} + \frac{b_2 y + 1}{\bar{Y}} \right) \).

Proof Lemma 5

If ISP chooses to use usage based pricing scheme then we set \( P_x > 0, P_Y > 0 \), dan \( P = 0 \). Then, Eq. (23) will be

\[
\left( U_{01} + a_1 \ln \frac{X_{11} + 1}{\bar{X} + 1} + b_1 \ln \frac{Y_{11} + 1}{\bar{Y} + 1} - P_{x1} - P_{y1} - 0 \cdot Z_1 \right) + \left( U_{02} + a_2 \ln \frac{X_{21} + 1}{\bar{X} + 1} + b_2 \ln \frac{Y_{21} + 1}{\bar{Y} + 1} - P_{x2} - P_{y2} \cdot 0 \cdot Z_1 \right) = \left( U_{01} + a_1 \ln \frac{X_{11} + 1}{\bar{X} + 1} + b_1 \ln \frac{Y_{11} + 1}{\bar{Y} + 1} - P_{x1} - P_{y1} \right) + \left( U_{02} + a_2 \ln \frac{X_{21} + 1}{\bar{X} + 1} + b_2 \ln \frac{Y_{21} + 1}{\bar{Y} + 1} - P_{x2} - P_{y2} \right) 
\]
(26)

Derive the Eq. (26) to optimize the consumer problem toward \( X_1, X_2, Y_1 \) and \( Y_2 \).

\[
\frac{a_1}{X_{11} + 1} - P_X = 0 \Leftrightarrow \frac{a_1}{X_{11} + 1} = P_X \text{ then } X_{11}^* = \frac{a_1}{P_X} - 1 
\]
(27)

\[
\frac{b_1}{Y_{11} + 1} - P_Y = 0 \Leftrightarrow \frac{b_1}{Y_{11} + 1} = P_Y \text{ then } Y_{11}^* = \frac{b_1}{P_Y} - 1 
\]
(28)

\[
\frac{a_2}{X_{21} + 1} - P_X = 0 \Leftrightarrow \frac{a_2}{X_{21} + 1} = P_X \text{ then } X_{21}^* = \frac{a_2}{P_X} - 1 
\]
(29)

\[
\frac{b_2}{Y_{21} + 1} - P_Y = 0 \Leftrightarrow \frac{b_2}{Y_{21} + 1} = P_Y \text{ then } Y_{21}^* = \frac{b_2}{P_Y} - 1 
\]
(30)

The supplier problem will be

\[
\max_{P_x,P_y} m(\bar{P_x} X_1 + \bar{P_y} Y_1) + n(\bar{P_x} X_2 + \bar{P_y} Y_2) 
= \max m(a_1 - P_X + b_1 - P_Y) + n(a_2 - P_X + b_2 - P_Y) 
\]
(31)

To maximize Eq.(31), ISP should lower the value of \( P_X \) and \( P_Y \). Since \( X_1, X_2, Y_1, \) dan \( Y_2 \) are restricted, then \( X_1^*, Y_1^* \), and \( Y_2^* \) cannot be more than \( \bar{X} \) and \( \bar{Y} \). For seeking the optimal price, first, we do the analysis on peak hours.

The supplier optimization problem will be as follows.

\[
\max_{P_x} m(\bar{P_x} X_1) + n(\bar{P_x} X_2) = \max m(a_1 - P_X) + n(a_2 - P_X) 
\]
(32)

To maximize Eq. (32) the supplier should lower the value of \( P_X \), so the best value of \( P_X \) cannot be greater than \( \frac{a_2}{\bar{X} + 1} \). In other words, if the supplier gives the prize below than we do not obtain maximum profit since the values of \( X_1^* \) and \( X_2^* \) are not greater than \( \bar{X} \) and the consumer demand will increase toward the decrement of the price. So, the best price \( P_X \) should be between \( \frac{a_2}{\bar{X} + 1} \) and \( \frac{a_2}{\bar{X} + 1} \). When the price is between that interval, high-end consumer demand will stay in \( \bar{X} \) will for low-end consumer will increase proportionally to the decrement of the price. Then, \( P_X \) and \( P_Y \) will be \( P_X = \frac{a_2}{\bar{X} + 1} \) and \( P_Y = \frac{b_2}{\bar{Y} + 1} \) and the maximum profit is \( (m + n) \left( \frac{a_2 x + 1}{\bar{X}} + \frac{b_2 y + 1}{\bar{Y}} \right) \).

Lemma 6: If ISP uses two part tariff scheme, the optimal price will be \( P_x = \frac{a_2}{\bar{X} + 1} \) and \( P_y = \frac{b_2}{\bar{Y} + 1} \).
\[ U_{o2} + a_2 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_2 \ln \frac{\bar{y} + 1}{y_{m+1}} - \frac{a_2 \bar{x}}{\bar{x} + 1} - \frac{b_2 \bar{y}}{\bar{y} + 1} \] and the maximum profit is \((m + n) \left( U_{o2} + a_2 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_2 \ln \frac{\bar{y} + 1}{y_{m+1}} \right).\)

**Proof of Lemma 6**

If ISP chooses to adopt two-part tariff pricing scheme, then it is set that \(P_X > 0, P_Y > 0\), and \(P > 0\). Then, Eq. (23) be:

\[
\left( U_{o1} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}} - P_X X_1 - P_Y Y_1 - P \right) + \left( U_{o2} + a_2 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_2 \ln \frac{\bar{y} + 1}{y_{m+1}} - P_X X_2 - P_Y Y_2 - P \right)
\]

(33)

First, to optimize the consumer problem, we need to derive Eq. (33) toward \(X_1, X_2, Y_1\) and \(Y_2\) so we have Eq. (27)-Eq. (30). After that, we will have \(P_X\) and \(P_Y\) whose value between \(\frac{a_1}{\bar{x}}\) and \(\frac{a_2}{\bar{x}}\). So, the best \(P_X\) is between \(\frac{a_1}{\bar{x}}\) and \(\frac{a_2}{\bar{x}}\). When the price is between that interval, high end consumer demand will remain the same in \(\bar{x}\) whereas the low-end consumer demand will stay proportional to decrement of the value. Then, \(P_X\) and \(P_Y\) will be \(P_X = \frac{a_1}{\bar{x}}\), \(P_Y = \frac{a_2}{\bar{x}}\) and \(P = U_{o2} + a_2 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_2 \ln \frac{\bar{y} + 1}{y_{m+1}} - \frac{a_2 \bar{x}}{\bar{x} + 1} - \frac{b_2 \bar{y}}{\bar{y} + 1}\). Assume that \(a_1 < m\) and \(b_1 < \frac{m}{m + n} < b_2\), then the maximum profit will be \((m + n) \left( U_{o2} + a_2 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_2 \ln \frac{\bar{y} + 1}{y_{m+1}} \right).\)

Then the type of financing schemes of flat fee and a two-part tariff generate huge profits equally in maximizing profit-making for the ISP (supplier) while the usage-based financing scheme obtain different value from the two schemes.

**C. Heterogeneous Consumer: High Demand and Low Demand**

**Lemma 7:** If ISP chooses flat fee, then the price paid will be \(P = U_{o2} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}}\) and the maximum profit will be \((m + n) \left( U_{o2} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}} \right)\).

**Proof of Lemma 7**

For flat fee scheme, we set \(P_X = 0, P_Y = 0\), dan \(P \geq 0\). If consumers choose to use the service then the maximum satisfaction level will be obtained by choosing composition level \(X_1 = \bar{X}_1, Y_1 = \bar{Y}_1\) or \(X_2 = \bar{X}_2, Y_1 = \bar{Y}_1\) with maximum satisfaction level \(U_{o1} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}}\) or \(U_{o2} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}}\).

So, ISP cannot charge the price greater than \(U_{o1} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}}\) to every high demand consumer and also to low demand consumer with \(U_{o2} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}}\). Since applying flat fee scheme, then ISP cannot differentiate the price between high demand and low demand consumers. So ISP should choose to charge \(U_{o1} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}}\) so the high demand consumer can apply the service with the price of \(U_{o2} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}}\) then high demand and low demand consumers can join the service. If we assume that \(m \left( U_{o1} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}} \right) < (m + n) \left( U_{o2} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}} \right)\), the best price that can be charged by ISP will be \(U_{o2} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}}\) for high and low demand consumers. So, the maximum profit obtained by ISP will be \((m + n) \left( U_{o2} + a_1 \ln \frac{\bar{x} + 1}{x_{m+1}} + b_1 \ln \frac{\bar{y} + 1}{y_{m+1}} \right)\).

**Lemma 8:** If ISP uses the usage based scheme then the price will be \(P_X = \frac{a}{x_1 + 1}\) and \(P_Y = \frac{b}{y_1 + 1}\) also the maximum profit is \(m \left( \frac{a x_1}{x_1 + 1} + \frac{b y_1}{y_1 + 1} \right) + n \left( \frac{a x_2}{x_2 + 1} + \frac{b y_2}{y_2 + 1} \right)\).

**Proof of Lemma 8**

Choosing the usage based scheme means that \(P_X > 0, P_Y > 0\), and \(P = 0\). First order derivative for both consumers are as follows.

\[
\frac{\frac{a}{x_1 + 1} - P_X = 0}{X_1 = a}{P_X} \Rightarrow X_1 = a P_X - 1
\]

(34)

\[
\frac{\frac{b}{y_1 + 1} - P_Y = 0}{Y_1 = b}{P_Y} \Rightarrow Y_1 = b P_Y - 1
\]

(35)

\[
\frac{\frac{a}{x_2 + 1} - P_X = 0}{X_2 = a}{P_X} \Rightarrow X_2 = a P_X - 1
\]

(36)

\[
\frac{\frac{b}{y_2 + 1} - P_Y = 0}{Y_2 = b}{P_Y} \Rightarrow Y_2 = b P_Y - 1
\]

(37)

The optimization problem of supplier will be

\[
\max m (P_X X_1 + P_Y Y_1) + n (P_X X_2 + P_Y Y_2)
\]

\[
= \max m \left( a_1 - P_X + b_1 - P_Y + n (a_1 - P_X + b_2 - P_Y) \right)
\]

(38)

since \(X_1, X_2, Y_1\) and \(Y_2\) are restricted then \(X_1^*, X_2^*, Y_1^*, Y_2^*\) dan \(Y_2^*\) will be \(X_1, X_2, Y_1, Y_2\). From Eq. (38), it can be seen that to maximize the profit, ISP should lower the value of \(P_X\) and \(P_Y\). Consider the peak hour time. Value of \(P_X\) cannot exceed \(\frac{a}{\bar{X}_1 + 1}\). If \(P_X\) is set to lower than \(\frac{a}{\bar{X}_1 + 1}\) then the profit cannot be optimal when \(X_1^*, X_2^*, Y_1^*\) and \(Y_2^*\) are not greater than \(X_1, X_2\) and also the consumer demand cannot increase due to value decrement. So, the best price for \(P_X\) should be between \(\frac{a}{\bar{X}_1 + 1}\) and \(\frac{a}{\bar{X}_2 + 1}\). When the price is between that interval, low demand consumer remains in \(\bar{X}_2\) and high demand
consumer will increase proportionally to price decrement. The, the best price for \( P_x \) will be \( a \frac{x}{x+1} \).

Also, for off-peak hour time, the price will be \( P_Y = b \frac{Y}{Y+1} \). So, \( P_X \) and \( P_Y \) will be \( P_X = \frac{a}{x+1} \) and \( P_Y = \frac{b}{Y+1} \) and the maximum profit is \( P = m(a \frac{x}{x+1} + b \frac{Y}{Y+1}) + n(\frac{a x}{x+1} + b \frac{Y}{Y+1}) \).

**Lemma 9:** If ISP uses two part tariff scheme, the price charged will be \( P_X = \frac{a}{x+1} \) and \( P_Y = \frac{b}{Y+1} \) and the maximum profit is \( P = m(a \frac{x}{x+1} + b \frac{Y}{Y+1}) + n(a \frac{x}{x+1} + b \frac{Y}{Y+1}) \).

Proof of Lemma 9:

If ISP choose to use two part tariff scheme, then \( P_X > 0 \), \( P_Y > 0 \), dan \( P > 0 \).

We use first order derivative to obtain Eq. (34)-Eq. (37). Then the supplier optimization problem will be

\[
\max_{P_X, P_Y} m(P_X X_1^* + P_Y Y_1^* + P) + n(P_X X_2^* + P_Y Y_2^* + P)
\]

\[
= \max_{P_X, P_Y} (a - P_X + b - P_Y + P) + n(a - P_X + b - P_Y + P)
\]

(39)

From Eq. (39) it can be seen that to obtain the maximum profit more than the value of \( P_X \) and \( P_Y \) should be as low as possible so the value of \( P \) remains large. So, \( X_1, X_2, Y_1, Y_2 \) are restricted then \( X_1^*, X_2^*, Y_1^*, Y_2^* \) will be \( \bar{X}_1, \bar{X}_2, \bar{Y}_1, \bar{Y}_2 \). \( P_X \) and \( P_Y \) will be between \( \frac{a}{x+1} \) and \( \frac{b}{Y+1} \). When the price is between that interval, the demand of the high demand consumer will stay at \( \bar{X}_2 \) and the demand of the high demand consumer will increase proportionally to value decrement. Then, the best price for \( P_X \) will be \( a \frac{x}{x+1} \). Then, for the analysis for non \( P_Y = \frac{b}{Y+1} \). In other words, \( P_X \) and \( P_Y \) will be \( P_X = \frac{a}{x+1} \) dan \( P_Y = \frac{b}{Y+1} \) with \( P = U_{02} + a \ln \frac{x}{x+1} + b \ln \frac{Y}{Y+1} - a \frac{x}{x+1} - b \frac{Y}{Y+1} \).

The maximum profit will be \( m(\frac{a x}{x+1} + b \frac{Y}{Y+1}) + (m + n) (U_{02} + a \ln \frac{x}{x+1} + b \ln \frac{Y}{Y+1}) \).

Lastly, from the analysis of high demand and low demand consumers, it can be seen that for each scheme, the maximum profit obtained are different. Due to the differences, we can conclude that two-part tariff achieves maximum(optimal) value.

IV. CONCLUSIONS

Based on the analysis conducted applying utility function as a function of bandwidth which decreases with increasing bandwidth by using three types of existing pricing schemes on homogeneous consumers and heterogeneous consumers we obtained optimal financing scheme on each consumer. Homogeneous consumer pricing schemes if ISPs choose to use flat fee or two-part tariff then ISP can obtain optimal profit.

Whereas in the case of heterogeneous customers which have two types, i.e., according to willingness to pay (willingness to pay) and the level of consumption (consumption level). For heterogeneous consumers with a desire to pay (high end and low end) ISP will gain maximum benefit when using flat fee or a two part tariff schemes. For heterogeneous consumers with different consumption levels (high demand and low demand) optimal scheme exists when ISPs choose to use two part tariff scheme of scheme.

The submitting author is responsible for obtaining agreement of all coauthors and any consent required from sponsors before submitting a paper. It is the obligation of the authors to cite relevant prior work in the paper. Authors of rejected papers may revise and resubmit.

ACKNOWLEDGMENT

The authors express their deep gratitude to the people who have helped the process of finding the solution analytically and to the university that has given the author the opportunity to speak up about the idea of the topic.

REFERENCES


